CARTOGRAMS AND TOPOLOGY

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ABSTRACT Highly generalized cartograms which are not rigidly bound to the geometry of the globe can often supply effective graphic solutions to problematical thematic situations, especially where non-metric object spaces are involved. While it is possible to reduce noise and to distribute information evenly over the map area, topological properties should be retained; some of these are described in this paper. In addition to uncontrolled cartograms, special distorting projections can now be employed in producing quantitative area cartograms.

We shall define a topological cartogram – or a cartogram for short – as a plane representation of a thematic space in which relations between points, lines, angles and areas are not rigidly bound to those found on the globe. The term topological map is sometimes applied,[1] and the French appellation anamorphose géographique[2] is self-explanatory in view of the above. The name topogram occurs in German use (e.g., Arnberger)[3] since many writers in German apply the term cartogram (Kartogramm) to all statistical maps with diagrammatic symbolization of data, although the *Multilingual Dictionary of Technical Terms in Cartography*[4] lists the German term 'Kartogramm' (under No. 823.17 (a)) but translates it into the English 'diagrammatical map,' whereas the English term cartogram (431.5 (b)) is defined as a synonym for the former. Some authors assert that no distinction should be made between map and cartogram (e.g., Imhof)[5] while others, although displaying cartographic illustrations of the type conforming to the definition above, simply disregard the distinction (see for example Dickinson).[6]

According to Thrower,[7] a cartogram is "a map which consciously distorts." Thus, in dealing with cartograms, we refer only to those graphics in which the distortions are introduced deliberately and even haphazardly, without numerical control, or mathematically. Mental maps, when given hard-copy form, seem to fall somewhere between unintentionally distorted maps and intentionally distorted cartograms; we shall disregard them in the present context.

Whereas in Western culture the scientifically-minded Greeks were the proponents of true and precise maps, having been the first to formalize cartographic projections, the Romans, with their practical and task-oriented approach, can perhaps be said to have been the (albeit unwitting) inventors of the cartogram with its deliberate and sometimes extreme distortions, as can be seen in the Peutinger Map described below in more detail.

Why should one, under certain circumstances, resort to the use of cartograms instead of maps? Let us look at the flow of information in a communications model (Figure 1). Here the encoder is the cartographer, and the transmitter is the

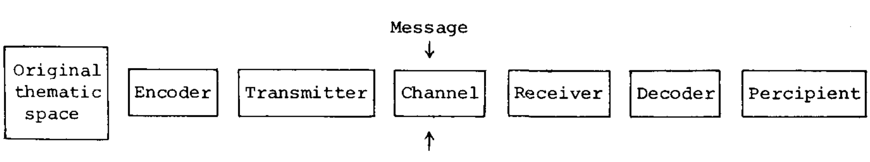

FIGURE 1. *Information flow diagram.*

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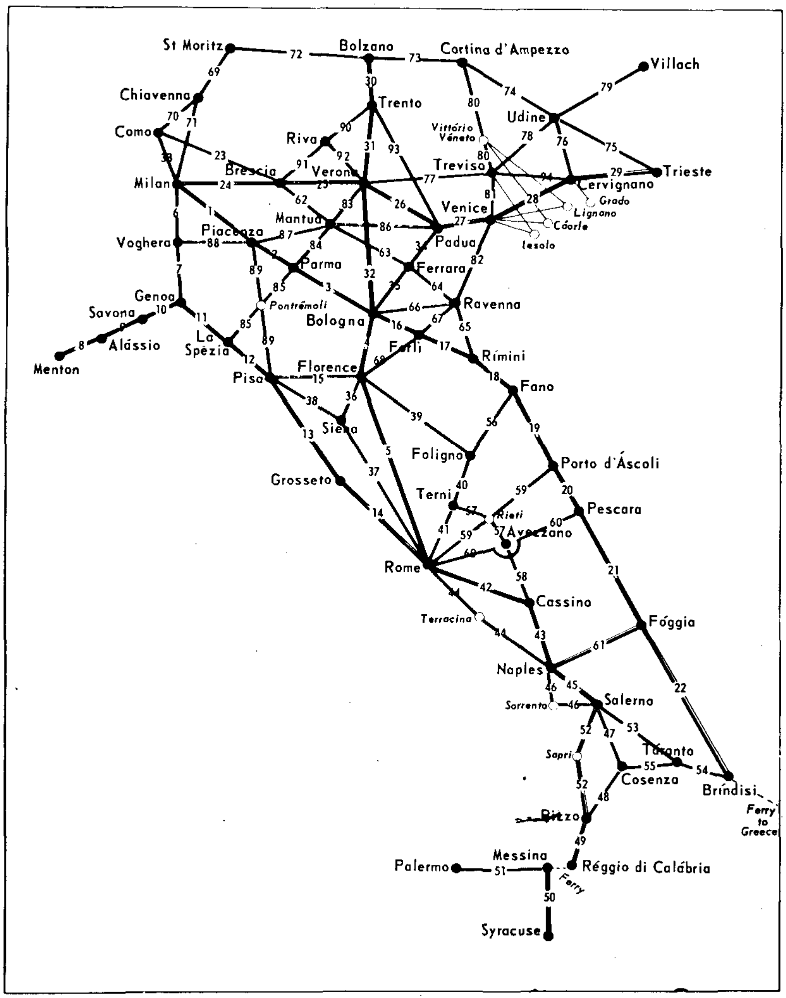

FIGURE 2. *Road cartogram without map background. The shape of Italy can be clearly discerned (Courtesy of The Automobile Association, United Kingdom).*

printing press. The channel is the actual map which transmits the spatial-geographic message, but which is also subject to graphic noise. The latter can be the result of overloading the map. Cartographic encoding, which is simply an alternative term for symbolizing, can also lead to noise if done inefficiently and without due regard to sound (noiseless?) cartographic principles. The percipient is the map reader; the receiver is his pair of eyes, while the decoder is his symbol-translating mind. What cartographic communication is really all about is ensuring that the losses in information incurred on the way between original thematic space and the percipient will be minimised, so that the percipient's image of the original thematic space will be as true to the latter as possible. But all graphic encoding entails loss of information, which is why the process is irreversible.

However, generalisation, and indeed over-generalisation, can assist in getting the main message across at the cost of data of marginal importance. Since, in our definition of a cartogram, we dispense with the rigid, spheroid-bound geometrical isomorphism of the conventional maps, we can streamline the cartogram to an extreme degree in two ways. Firstly, we can eliminate all detail not directly related to the main subject of the cartogram. Indeed, some cartograms go so far as altogether doing away with the background map, an act justified only if the spatial distribution of the subject at least hints strongly at the shape or outline of the geographical region covered. A road cartogram of Australia, and especially of the Northern Territory and Queensland, would hardly do this; one of Italy or Britain does it admirably (Figure 2). Secondly, lines of the thematic variable can be greatly smoothed and even completely straightened. One may, of course, question the justification of this step. But since our aim is to reduce graphic noise and increase the efficiency of the cartogram as a channel for spatial information, we may indeed be improving the reconstruction of the original thematic space in the mind of the map reader by making decoding easier. The graphic and plastic arts, too, do this sort of streamlining, for example in Impressionism. But it would never do in an anatomic atlas which may be used for probing the more sensitive parts of our insides during surgery; similarly, neither could one dispense with locational accuracy in topographic, navigational, orienteering or cadastral maps.

SOME TOPOLOGICAL CONCEPTS

Cartograms can be classified into point, line and area graphics, in common with conventional maps. Since these three concepts, although dressed in different terminology, are among the elements of topology, we shall now briefly review some of the basics of the latter. Topology was established as a branch of mathematics by Leonard Euler in the 18th Century, but the name was given only some 100 years later. Topology uses the terms map and mapping (the latter for certain transformations) which make it sound familiar to the cartographer. It deals, among others, with networks composed of points called nodes or vertices; with lines connecting them, called arcs, routes or edges; and with regions or faces generated by the latter (Figure 3). Nodes can be connected or unconnected to each other, like farms connected or unconnected through a road net; and arcs can connect all or only some nodes in a net, like the roads in the example above. Regions can be closed, if entirely surrounded by arcs, or open, if not.[8] Topology has been called 'rubber sheet geometry.' For example, Chinn and Steenrod,[9] who also convincingly illustrate the assertion (p. 58) that a topologist is a mathematician who cannot tell the difference between a donut and a cup of coffee. Not the metric locations, lengths, areas and angles of a network concern the topologist, and from a topological point of view a straight line and all possible curved lines between two given nodes in a given space are equivalent, as are regions. This can be demonstrated by drawing the network on a sheet of rubber; as long as one does not tear it, no amount of stretching will change its topological properties. The same is true not only of surfaces but for multi-dimensional spaces. The present writer uses rubber toy balloons with graticules drawn on their surface to demonstrate the

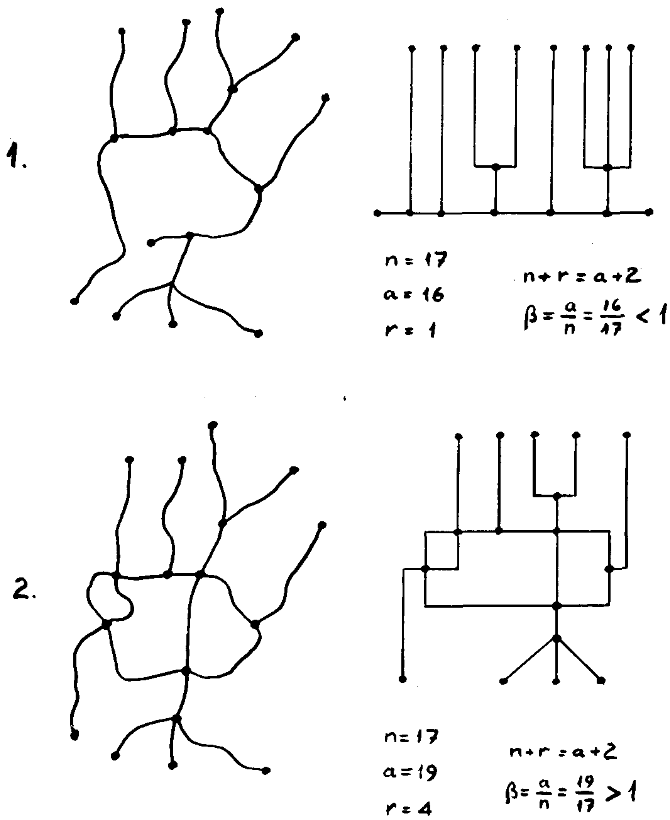

FIGURE 3. *Topological networks. See text for explanations.*

topological properties of cartographic projections by inflating, deflating and suitably stretching the balloons.

A basic topological relationship states that in any network the combined number of nodes n and regions r (including open regions) is always greater by 2 than the number of arcs a, i.e.,

1
$$n + r = a + 2$$

In view of the above, Sanson-Flamsteed's Sinusoidal projection and Lambert's Cylindrical Equal-Area projection are topologically equivalent (and tough luck for the cartography student that geometrically they are not); but the Stereographic, in which one point, namely the projection centre, is always missing, is topologically different. The Orthographic and the Gnomonic projections can both represent a hemisphere, the former in practice because it is closed, while the latter in theory only, since it is open, with the hemispheral perimeter missing.

Order and contiguity are other topological concepts, as is the connectivity index

2
$$\beta = \frac{a}{n}$$

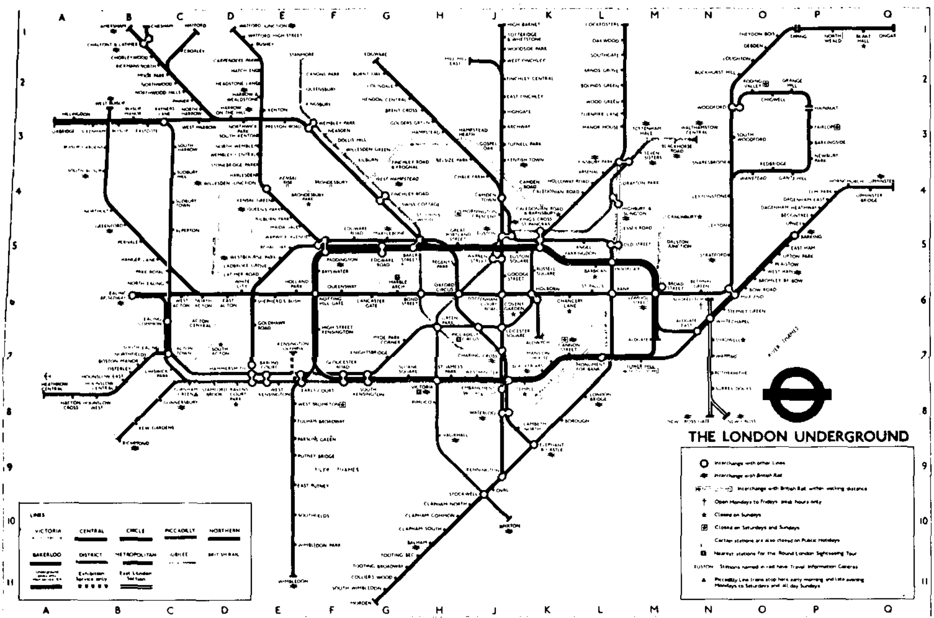

FIGURE *4*. *London Underground Lines (London Transport Board).*

If the number of arcs is equal to or greater than that of the nodes, and thus $\beta \geqslant 1$, a closed region with a route around it must exist. This is of some interest to travellers on underground railways. Simplified and highly generalised maps of subways are among the best known cartograms, and that of the London Transport Board's Underground Lines is a model of topological clarity (Figure 4). In both London's and Moscow's (to name but two) Circle Line, a = n (where n represents the number of stations and thus $\beta = 1$. However, in Sydney's City Circle Line (Figure 5) which is open at Central Station, $\beta = 6/7$, since there exists no Central-to-Central arc! If the City Circle Line's cartogram were completely circular (and thus topologically wrong), unwary Underground fans who, like the present author, just love travelling in subways in circles, although this stimulates their mental processes to do the same, might find themselves in difficulties (or when in Sydney, in Sydenham).

Sydney's City Circle's diagrammatic map can also represent another topological concept, namely the König number, which is a measure of centrality of a node. This is the maximum number of arcs traversed in travelling from a given node to any of the others by the shortest routes, and is usually seen much clearer in a cartogram than in a map. The shortest way from Sydney's Museum station to Town Hall traverses 4, not 2 arcs, as would appear from a closed circle cartogram. And the diameter of this peculiar 'circle' (the diameter being the minimum number of arcs to be traversed between the two most distant points in the network) is 6 (Central-to-Central) and not 3 (Central to Circular Quay) as might be expected in a closed network with the same number of stations and connectivity index of 1.

London's Circle Line has 27 stations or nodes, and its diameter is 13, the number of stations one travels say from Paddington to Mansion House – in either direction.

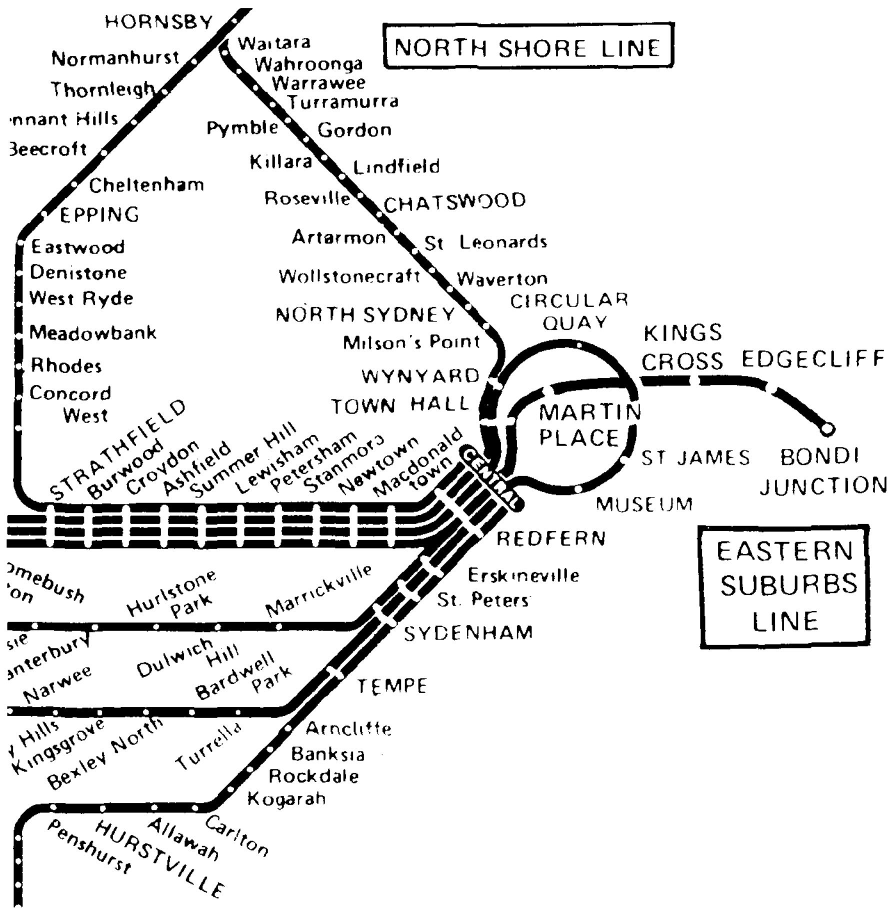

FIGURE 5. *Sydney Circle Line (State Rail Authority of New South Wales).*

Topology has many more indices for networks, but we shall mention only one more, the arc or route matrix. This is a square array with a list of all nodes in proper order along both horizontal and vertical axes. A figure 1 at the respective intersection between nodes indicates that they are connected by a route, while a 0 denotes a lack of route. The node with the largest number of 1's is the most 'central' one. This matrix is symmetrical (unless we wish to distinguish between one-way and two-way routes), and therefore only the closed diagonal matrix (i.e., the half matrix including the diagonal) is required. This is, of course, the well-known form of many conventional road distance charts accompanying some maps and timetables.

GUIDELINES FOR CARTOGRAMS

The cartographer confronted with a problem requiring severe generalization and simplification, or one that makes the extreme stressing of only the vital character-

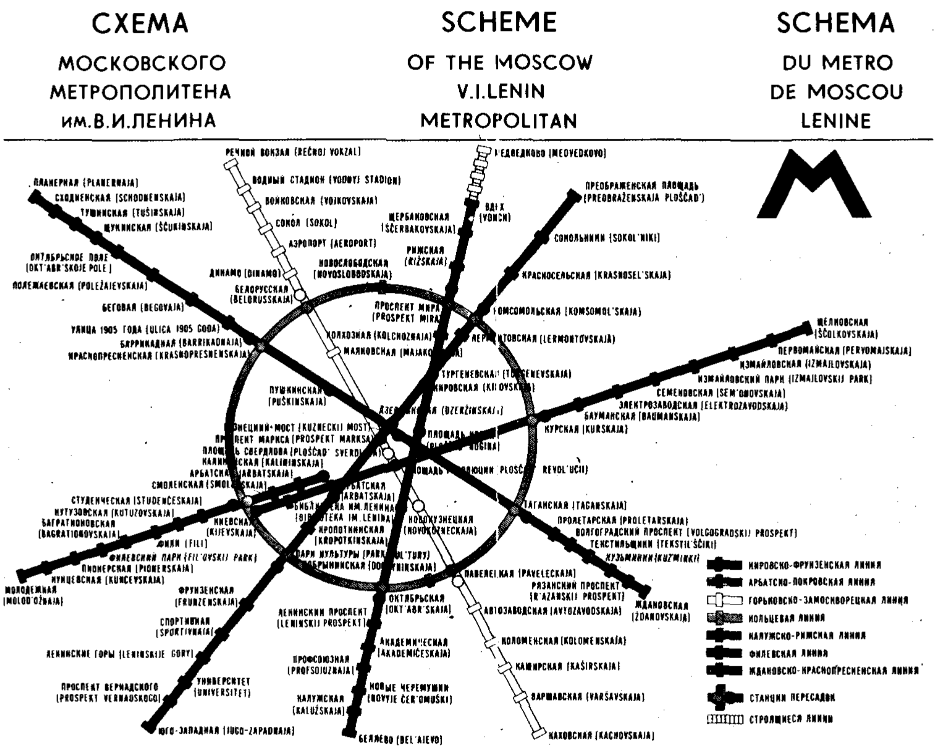

FIGURE 6. *Moscow's Metropolitan Lines.*

istics of the subject desirable or necessary, may thus well resort to using a cartogram. And if a non-Euclidean or non-metric object space is to be depicted, this holds doubly true. What, then, are the guidelines for producing an acceptable cartogram?

After being presented with the subject the cartographer must, in the first place, decide on the dimensionality of the theme to be stressed, whether point, line or area, and select a suitable graphic mode. All, or nearly all, detail not related directly to the main subject must now be eliminated by the map editor, who must then ensure that the number of thematic nodes, arcs and regions in the cartogram – as the case may be – correspond to those of the original object space. Having selected them he has to connect the nodes by simplified or streamlined arcs so that the main characteristics of the spatial distribution such as proximity and approximate directions are not only preserved but stressed. But – and here lies the crux of the matter – this has to be done so that the topological properties outlined above are indeed retained invariantly. This refers primarily to open and closed regions, and formula 1 above should be used as a test. The diameter of the network should be checked and found equal to that of the Euclidean map. Other indicators, such as the König number, are more of an aid to the map reader than to the producer of the cartogram.

Considerations of the map-reading ability of the potential user population of

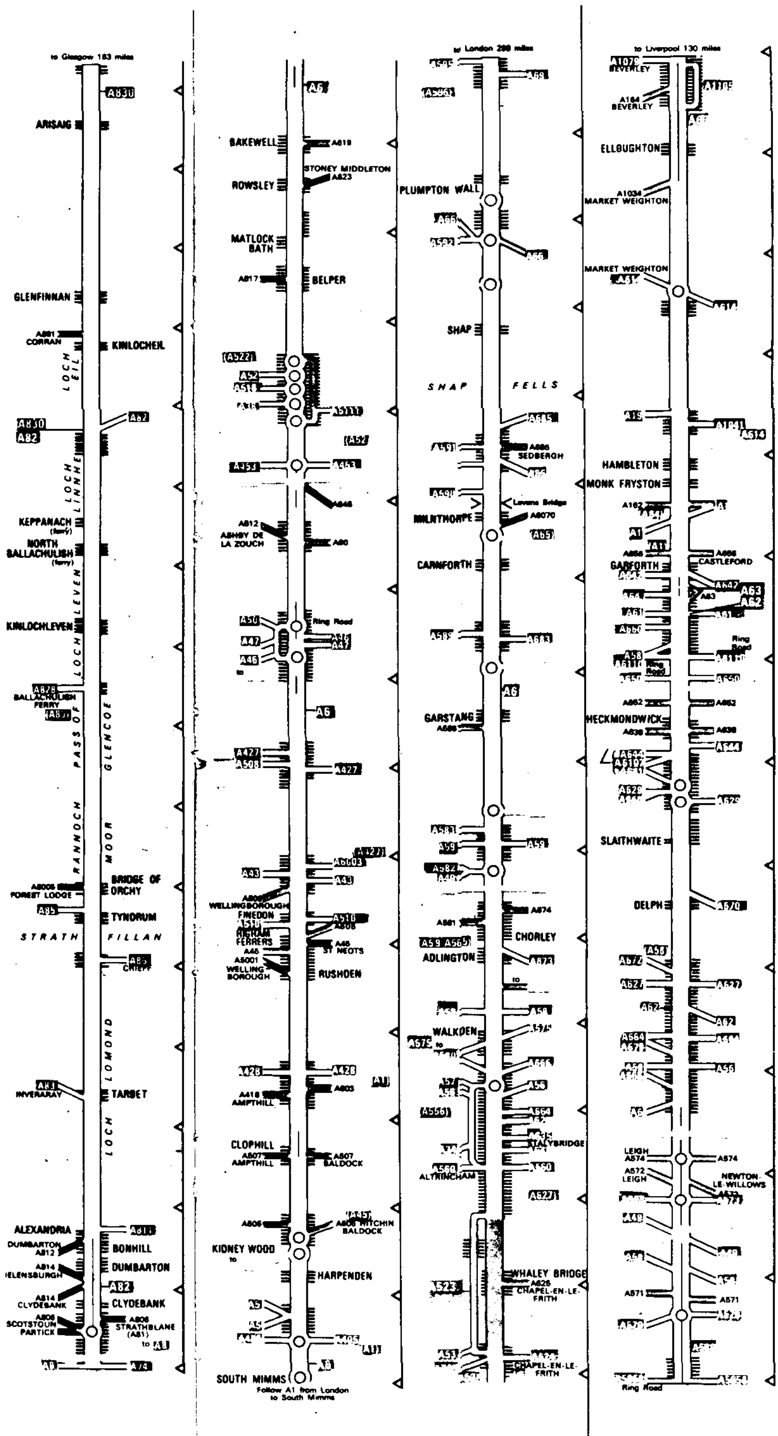

FIGURE 7. *Straight-lined road strip map (Courtesy of The Automobile Association, United Kingdom).*

a map sometimes dictate the degree of simplification needed, and thus may make the use of a cartogram desirable. The London Transport Board must have had the 'weakest link' in the range of their customers in mind when they had the famous Underground cartogram designed. The comparison of this with the topographic map of London and its Underground lines illustrates this very well (both maps are displayed in many of London's tube stations side-by-side). It also shows the principle of eliminating nearly all irrelevant detail, thus reducing graphic noise. In London's Underground case only the river Thames is retained from among the topographic features, and this in streamlined form only.

True orientation can – and often should – be dispensed with. London's Northern Line topographically hardly ever points north, but the cartogram represents much of it in a straight line in this cardinal direction, relieving the reader of the unnecessary mental burden of variable direction. Similarly, Moscow's Koltsevaya Linia – Circular Line – is depicted as a true circle, although being far from running in a real geometrical circle (Figure 6). Further good examples are road maps and atlases in which main road segments are drawn as straight lines from bottom to top of the sheet irrespective of true direction, with other roads branching off on both sides. For ease of reading by drivers in both directions of travel, some road atlases show each main road segment twice on adjacent pages, once for each direction (see Figure 7).

Finally, the nodes and arcs should be placed on the cartogram so as to maximize the utilization of available space. This often entails a step which is out of reach of the 'orthodox' cartographer, namely providing the reader with nearly homogeneous density of information. Whereas on a true planimetric basis some features may be so crowded together as to require a large map scale (which will then lead to relative emptiness in other parts of the map), the cartogram can – and should – utilize the actual sheet of paper in a much more efficient form, with information spread evenly. It should be unnecessary at this stage to stress that this principle holds true only for non-metric or non-topographic information. This approach of spatial entropization of information – the irreversible homogenization of data density – is perhaps the nearest approximation in cartography to the original use of the term entropy in thermodynamics. But while an increase of entropy reduces information content (see e.g., Knöpfli)[10] we find that in the case of a more or less homogenized-density cartogram total information may indeed be reduced, but the remaining data content is enhanced in clarity, leading to better readability and thus partly making up for the loss of information incurred by generalization.

THE ROAD DISTANCE CARTOGRAM

A useful application of cartograms is that showing actual road distances between any two points in the form of straight line segments of the respective length, instead of air distances between the same points. Using data from a motoring guide of, say, Australia, results in a cartogram which shows Australia stretched, in relation to the good old well-known shape, not east-to-west but in a north-easterly direction in New South Wales and Victoria. The remote places of the Northern Territory and the York Peninsula would, of course, be way out!

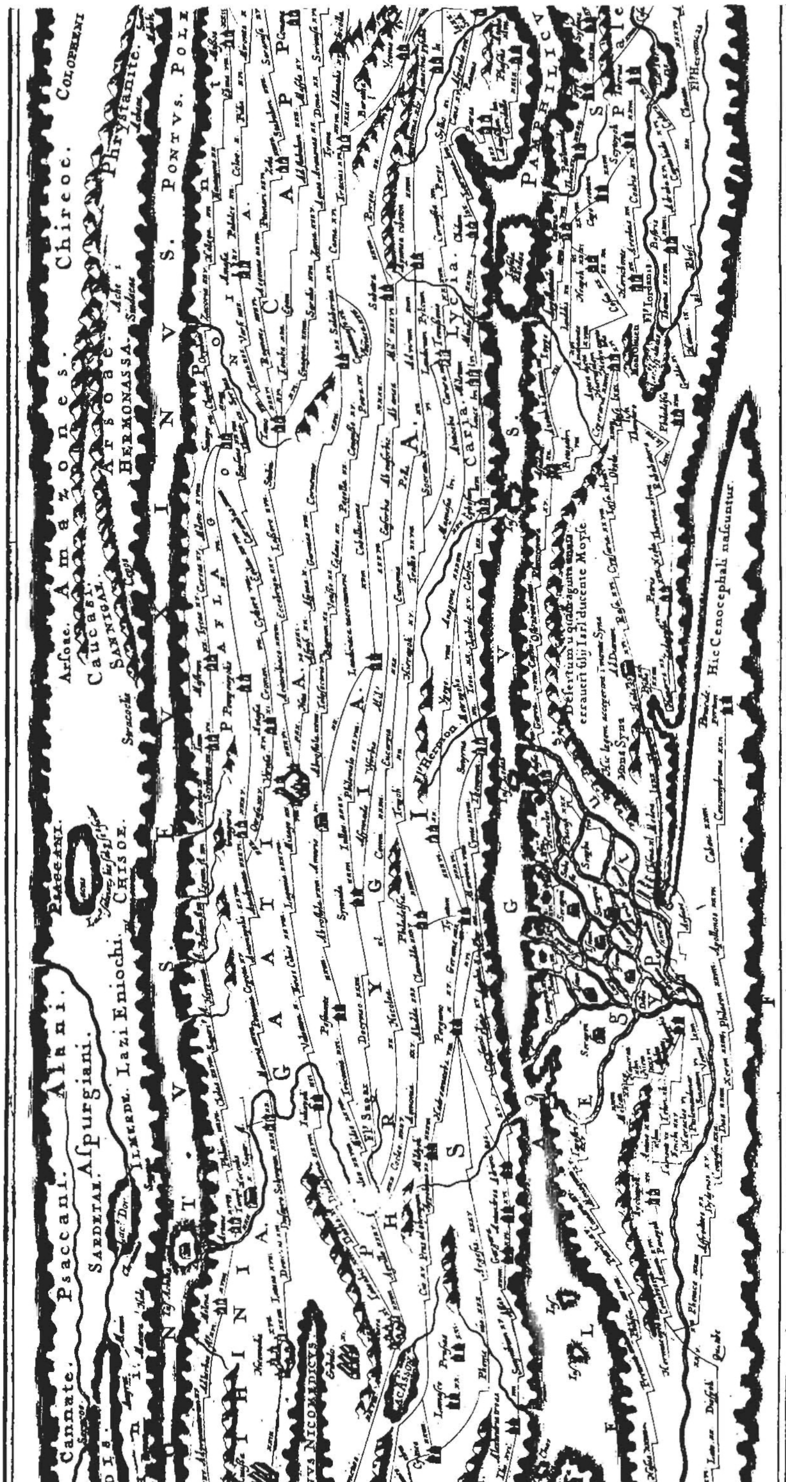

FIGURE 8. *The Peutinger Map. Stepped lines represent Roman roads with staging posts and distances in Roman miles.*

A road distance matrix always has a graphic solution if distances are measured from one given node (i.e., the matrix is a row or column matrix), all other places usually being shown in their true direction relative to this node (azimuthal equidistant cartogram!); distances between other nodes will generally be distorted. If distances between other places are considered, no graphic solution in the plane may exist in certain cases.

I should like to close this section of the present article with mention of one of the oldest and finest examples of cartograms, namely the Peutinger Map[11] produced in the 12th Century apparently as a copy of a lost Roman map dating from about the year 365 A.D. (Figure 8). It shows the network of paved roads of the Roman Empire – with 101,000 km of annotated roads! – as straight red lines with little steps in them to denote staging posts or stations, with Roman numerals expressing the length of each stage in Roman miles (milia passuum). The background is a map completely compressed in the north-south direction, reducing the Black and Mediterranean Seas, not relevant to the subject of the map, to mere wavy lines. This was done in order to have the map on a long and relatively narrow strip of parchment, some 35 cm wide and 5 m long, easily rolled up into a scroll for travel. There seems to be nothing new under the sun!

CONTROLLED CARTOGRAMS

Up till now we have discussed cartograms with mathematically uncontrolled distortions. Cartograms are often employed to represent non-metric subject spaces, which, for the purpose of this paper, can be defined as being composed of two (or three, as the case may be) geometrical dimensions expressed in units of length, with one or more thematic dimensions superimposed. In a simple example the X and Y dimensions might be rectangular coordinates, the Z being a cost, distance, time or even desirability function of X and Y such as desirability of environmental or living conditions. Indeed, man (and woman!) often thinks in non-metric concepts in relation to space, e.g., in financial or economic situations – or in matters of the heart, where mental distance to one's girl- or boyfriend is usually quite non-metric.

One of the first attempts at quantitative representation of a non-metric space was made by Hägerstrand,[12] for whom Edgar Kant devised the, by now, quite well known Logarithmic Azimuthal Projection, expressing a migrant's impression of distance from his native Sweden. This was a one-off production. The Falk parabolic town plans in the German Federal Republic have a scale which radially decreases from the map centre outwards, for example from 1 : 10,000 to 1 : 25,000. This was done in order to show in a continuous representation both large-scale detail in the dense city centre, and small-scale large suburban areas, without recourse to disjointed inset maps. These maps are produced by mechanical means, not with the aid of a projection. The present author proposed the Hyperbolic projection described in the *International Cartographic Yearbook*, 1975;[13] this is processed by computer, with interactively or 'instantly' variable projection centre, scale at map centre and scale friction function, i.e., the rate of radial scale decrease (Figure 9). This can produce town (and other) maps on an automated plotter. If

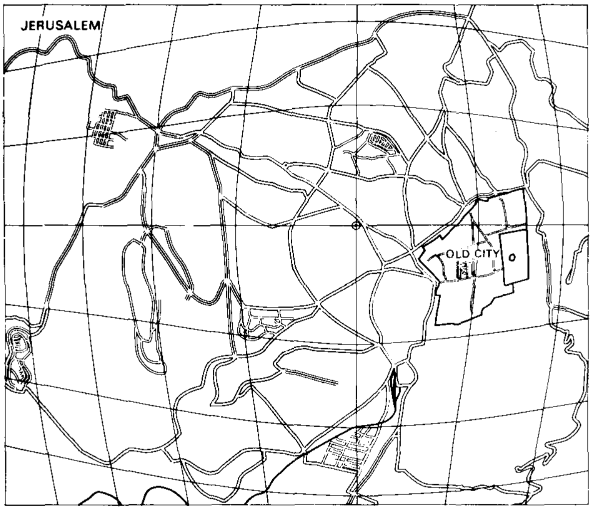

FIGURE 9. *Hyperbolical-scale generalized road cartogram of Jerusalem drawn by automated plotter (N. Kadmon).*

applied to Sydney, it would enable the writer to get from Kensington and the University of New South Wales into the City without having to carry a heavy road index atlas or, alternatively, a small-scale map of the whole Metropolitan area, rather similar in scale to what Governor Arthur Phillip must have used in 1788.

These computed cartograms, retaining all topological properties as well as true angles at the centre (the 'focus'), are azimuthal cartograms. One interesting application is in the transportation field. If the average traffic speed in the City centre and its increase with radial distance (with decreasing congestion) is known, map scale can be made to represent driving time, e.g., in minutes/km, and average driving times in minutes can be read directly off the map. The computer/plotter can easily produce separate maps for different times such as mid-day conditions, rush hours etc. Frustrated city drivers, please note! However, this is true only in radial directions from some central point.

THE POLYFOCAL PROJECTION

The next logical step was devising a projection overcoming this single-focus limitation, with an arbitrary and unlimited but finite number of foci, each with its own scale, proportional to some thematic variable (such as time, transportation speed, desirability, etc.) and its individual scale friction (decrease) function around it. The scale at any other point was made equal to the sum of the influences of all adjacent foci on this point. A well-known geographer once wrote that such a projection is impossible to achieve.[14] Shlomi and Kadmon produced one and named it Polyfocal (described in *The Cartographic Journal*, June 1978)[15] with computer program POLYMAP producing the maps on a plotter.

We now turn to area cartograms. The value of a choropleth map is well established, but it has the disadvantage that the areas of its statistical-territorial subdivisions are proportional to metric land areas, and not to any thematic values.

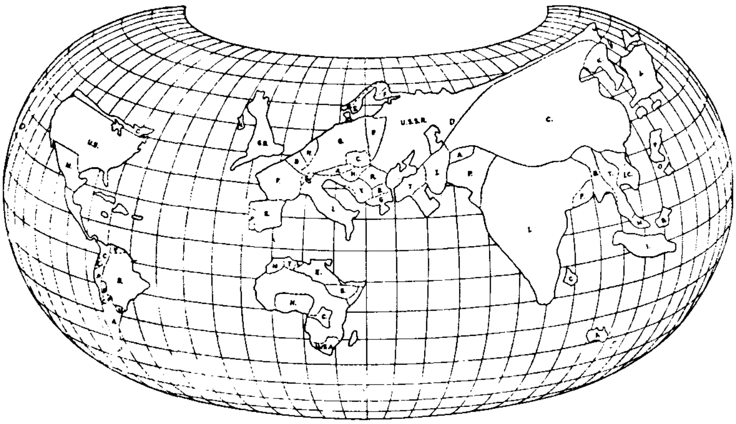

FIGURE *10*. *Woytinski's value-by-area Armadillo 'projection'.*

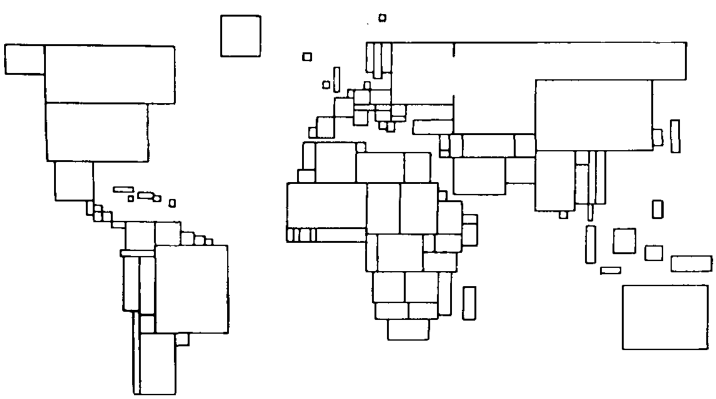

FIGURE *11*. *Countries of the world-value-by-rectangular-area (after Cuénin).*

Therefore, some substitute had to be found for expressing the latter, and tints and colours are most frequently used in this role, often quite unsatisfactorily. Sometimes height in a perspective view is used. In the past, many attempts were therefore made to produce so-called value-by-area maps of non-metric spaces, such as Woytinski's 'Armadillo surface' (Figure 10), again a manual one-off affair.[16] Better known are rectangle cartograms, in which statistical units, e.g., countries, were simulated by rectangles as well as possible, the areas of which were proportional to the respective values of the thematic variable[17] (see Figure 11). Each such cartogram had to be made individually by hand, and often the map resulted in a disjointed array of rectangles or other shapes. The topological properties of contiguity and connectivity were not retained in relation to reality. Williams[18] has produced a shape-retaining value-by-area cartogram by computer, though without retaining contiguity of the territorial units. One of the possible

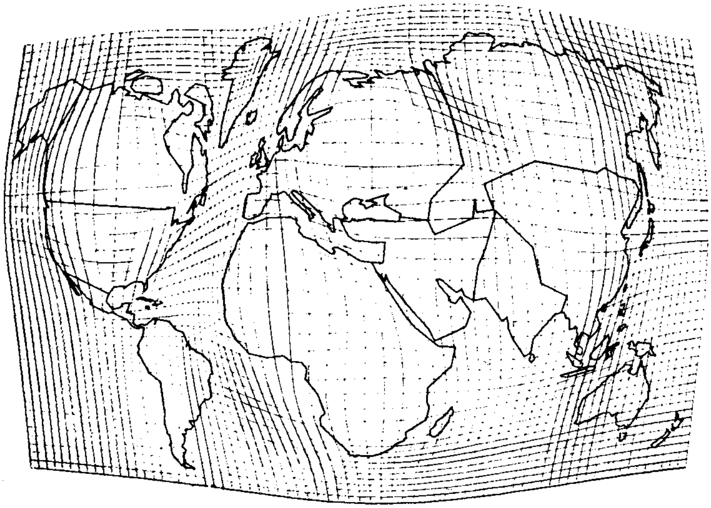

FIGURE 12. *The world's continents and blocs drawn by automated plotter on the Polyfocal projection with areas proportional to a statistical value (N. Kadmon and E. Shlomi).*

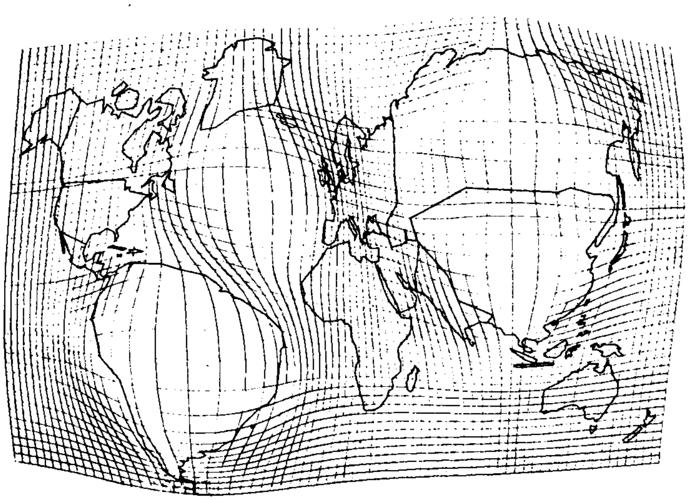

FIGURE 13. *The Polyfocal projection with different statistical values.*

applications of the Polyfocal projection mentioned above is that it can, by a process of iterations, plot the countries not only with area proportional to statistical values, but as a continuous surface, though not conformal and with distortions in linear and angular measures. However, it retains the topological features, including contiguity and connectivity (Figures 12, 13). And, of course, any small-scale map of an extensive area suffers from distortions of some kind. If now the area of each country or other statistical-territorial unit represents one variable, say crude oil production, then colour can be added, if desired, to express a second, such as oil

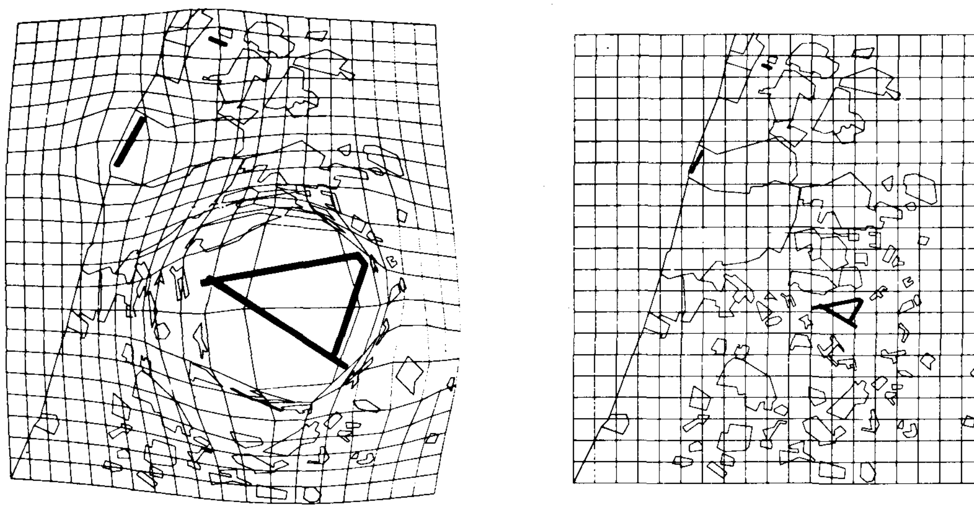

FIGURE *14*. *Noise pollution from three airports in the Tel-Aviv region. Automated cartogram plotted with the aid of the Polyfocal projection (N. Kadmon).*

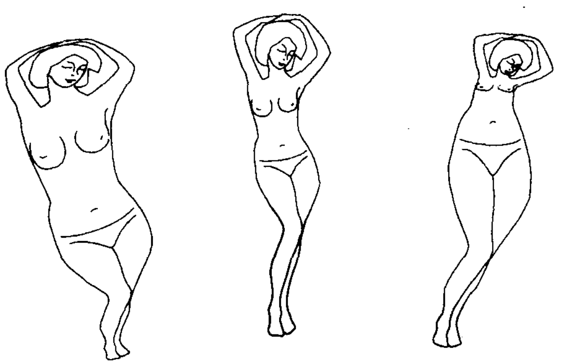

FIGURE *15*. *Three Polygirls. The central figure was twice subjected to the Polyfocal transformation with different parameters. Facial expression remains nearly unchanged.*

consumption, and we can come up with a multi-thematic cartogram. Another application now used is in the field of mapping the environment, with local noise pollution from three airports in the Tel-Aviv region being represented by local map scale[19] (see Figure 14). Griffin[20] manually produced a topological transformation for one map of Adelaide census data, which required 20 hours of work. POLYMAP would do this in 3–5 minutes.

Other uses may be devised, and some were. We now will adopt Griffin's view that a cartogram can be used "as a device to shock ... by the graphic display of some unsuspected spatial peculiarities." Thus, transforming the contours of a (preferably shapely) human body by the Polyfocal projection shows that whatever is done to its (i.e., the body's) proportions, facial expression remains pretty much the same, a point which is of interest to students of art and psychology (Figure 15).

Moreover, man's and woman's shape in all its diversity can now be classified in a quantitative matrix as simple or as complex as one may wish – but why, indeed, should we wish to quantify beauty?

NOTES

[1] P. Haggett, 1970. *Locational analysis in human geography.* Edward Arnold, London, p. 237.
[2] R. Cuenin, 1972. *Cartographie générale.* Tome 1, Notions générales et principes d'élaborations. Éditions Eyrolles, Paris, p. 310.
[3] E. Arnberger, 1966. *Handbuch der thematischen Kartographie.* Franz Deuticke, Wien, p. 63.
[4] International Cartographic Association, Commission 3, 1973. *Multilingual dictionary of technical terms in cartography.* Franz Steiner Verlag, Wiesbaden.
[5] E. Imhof, 1972. *Thematische Kartographie.* Walter de Gruyter, Berlin, p. 114.
[6] G.C. Dickinson, 1973. *Statistical mapping and the presentation of statistics.* 2nd ed., Edward Arnold, London, p. 66.
[7] N.J.W. Thrower, 1972. *Maps and Man.* Prentice-Hall, Englewood Cliffs, p. 149.
[8] There exist many introductory texts in topology. See e.g., M.A. Armstrong, 1979. *Basic topology.* McGraw Hill (UK), Maidenhead; or D.W. Blackett, 1976. *Elementary topology.* Academic Press, New York and London (the latter brings some interesting cartographic applications).
[9] W.G. Chinn and N.E. Steenrod, 1966. *First concepts of topology.* Random House, Stanford, p. 57.
[10] R. Knöpfli, 1978. Information, Modell und Karte. *International Yearbook of Cartography,* vol. 18, pp. 65–87. See also R. Knöpfli, 1975. Information, Karte und Flugbild. Mensuration, Photogrammetrie, Génie rural, 1–75, pp. 65–68.
[11] K. Miller, 1888. *Die Weltkarte des Castorius, genannt die Peutinger' sche Tafel.* Ravensburg. See also J. Elster and N. Kadmon, 1970. The Tabula Peutingeriana, in *Atlas of Israel.* 2nd ed., Survey of Israel and Elsevier Publishing Company, Tel-Aviv, Sheet 1/2.
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[14] P. Haggett, 1972. *Geography – a modern synthesis.* Harper and Row, p. 98.
[15] N. Kadmon and E. Shlomi, 1978. A polyfocal projection for statistical surfaces. *The Cartographic Journal,* vol. 15, no. 1, pp. 36–41.
[16] R. Murphy, 1971. *An introduction to geography.* 3rd ed., Rand McNally, Chicago, p. 64.
[17] R. Cuenin, 1972. *Cartographie générale.* Tome 1, Notions générales et principes d'élaboration. Editions Eyrolles, Paris, pp. 309–310.
[18] A.U. Williams, 1978. Interactive cartogram production on a microprocessor graphics system. *Proceedings of the American Congress on Surveying and Mapping,* Washington, D.C., fig. 1, p. 427.
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RESUME Il est souvent très difficile de représenter graphiquement des thèmes complexes, surtout lorsqu'on a affaire à des données qui ne se mesurent pas concrètement. Les cartogrammes de généralisation s'avèrent souvent une solution efficace car ils n'ont pas à tenir compte des formes vraies du globe. Si on peut réduire les distorsions et distribuer l'information uniformément sur la surface de la carte, la topologie doit toutefois être respectée. L'article traite de certains de ces aspects. En plus des cartogrammes non contrôlés, on peut maintenant produire des cartogrammes de phénomènes quantitatifs à l'aide de projections de distorsion.

ZUSAMMENFASSUNG Stark generalisierte Kartogramme, die nicht streng der Geometrie des Globus folgen, können oft effektive graphische Lösungen für problematische thematische Situationen bieten, besonders wenn nichtmetrische Objekträume mitspielen. Während es moglich ist, Informationen gleichmässig über die Kartenfläche zu verteilen, sollten topologische Eigenheiten erhalten bleiben, von denen einige in diesem Aufsatz besprochen werden. Ausser unkontrollierten Kartogrammen können auch verzerrte Abbildungen benutzt werden, um quantitative Flächenkartogramme herzustellen.

RESUMEN Los cartogramas altamente generalizados que no están rigidamente fijados por la geometría de globo frecuentemente pueden dar soluciones gráficas efectivas a situaciones temáticas problematicas, especialmente cuando se trata de espacios de objetivos no métricos. Aunque se posible reducir 'ruido' y distribuir información uniformemente sobre el área de mapa, las propiedades topológicas deben retenerse; se describen algunos de ellos en este trabajo. Además de cartogramas no controlados, ahora se puede utilizar proyecciones distorsionados especiales para la producción de cartogramas de areas cuantitativas.